

Hong Kong Mathematics Olympiad (2007 – 2008)

Heat Event (Individual)

香港数学竞赛 (2007 – 2008)

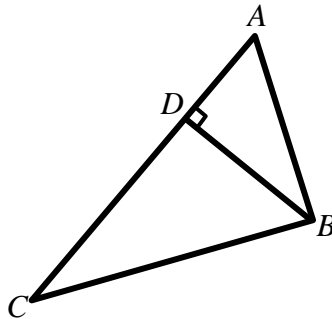
初赛项目(个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 如图一， $ABC$  为一个三角形且  $AB = 13\text{cm}$ 、 $BC = 14\text{cm}$  及  $AC = 15\text{cm}$ 。 $D$  为  $AC$  上的一点使得  $BD \perp AC$ 。若  $CD$  比  $AD$  长  $X\text{cm}$ ，求  $X$  的值。

In Figure 1,  $ABC$  is a triangle,  $AB = 13\text{cm}$ ,  $BC = 14\text{cm}$  and  $AC = 15\text{cm}$ .  $D$  is a point on  $AC$  such that  $BD \perp AC$ . If  $CD$  is longer than  $AD$  by  $X\text{cm}$ , find the value of  $X$ .



图一

Figure 1

2. 已知梯形  $PQRS$  的边长分别为  $PQ = 6\text{cm}$ 、 $QR = 15\text{cm}$ 、 $RS = 8\text{cm}$  及  $SP = 25\text{cm}$ ，并有  $QR \parallel PS$ 。若  $PQRS$  的面积为  $Y\text{cm}^2$ ，求  $Y$  的值。

Given that a trapezium  $PQRS$  with dimensions  $PQ = 6\text{cm}$ ,  $QR = 15\text{cm}$ ,  $RS = 8\text{cm}$  and  $SP = 25\text{cm}$ , also  $QR \parallel PS$ . If the area of  $PQRS$  is  $Y\text{cm}^2$ , find the value of  $Y$ .

3. 已知  $x_0$  及  $y_0$  为正整数且满足方程  $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$ 。若  $35 < y_0 < 50$  及  $x_0 + y_0 = z_0$ ，求  $z_0$  的值。

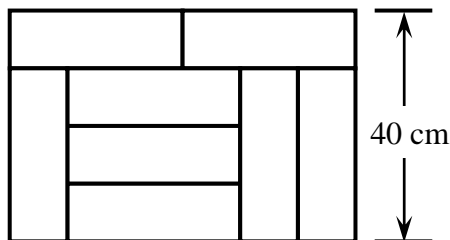
Given that  $x_0$  and  $y_0$  are positive integers satisfying the equation  $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$ . If  $35 < y_0 < 50$  and  $x_0 + y_0 = z_0$ , find the value of  $z_0$ .

4. 设  $a$ 、 $b$ 、 $c$  和  $d$  为实数。若  $\frac{a}{b} = \frac{1}{2}$ ,  $\frac{b}{c} = \frac{3}{2}$ ,  $\frac{c}{d} = \frac{4}{5}$  及  $\frac{ac}{b^2 + d^2} = e$ , 求  $e$  的值。

Let  $a, b, c$  and  $d$  be real numbers. If  $\frac{a}{b} = \frac{1}{2}$ ,  $\frac{b}{c} = \frac{3}{2}$ ,  $\frac{c}{d} = \frac{4}{5}$  and  $\frac{ac}{b^2 + d^2} = e$ , find the value of  $e$ .

5. 如图二，利用 8 个相同的小长方形能拼出一个大的长方形。已知在大长方形中较短的边长为 40 cm。若小长方形的面积是  $A \text{ cm}^2$ , 求  $A$  的值。

In Figure 2, the large rectangle is formed by eight identical small rectangles. Given that the length of the shorter side of the large rectangle is 40 cm and the area of the small rectangle is  $A \text{ cm}^2$ , find the value of  $A$ .

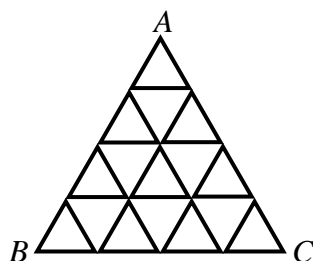


图二

Figure 2

6. 在图三中， $\triangle ABC$  为等边三角形。它由多个相同的等边三角形组成。若图中共有  $N$  个等边三角形，求  $N$  的值。

In Figure 3,  $\triangle ABC$  is an equilateral triangle. It is formed by several identical equilateral triangles. If there are altogether  $N$  equilateral triangles in the figure, find the value of  $N$ .



图三

Figure 3

7. 设  $r$  为方程  $\frac{4}{y+1} + \frac{5}{y-5} = -\frac{3}{2}$  的较大实根。求  $r$  的值。

Let  $r$  be the larger real root of the equation  $\frac{4}{y+1} + \frac{5}{y-5} = -\frac{3}{2}$ . Find the value of  $r$ .

8. 设  $x$  为有理数及  $w = \left| x + \frac{2007}{2008} \right| + \left| x - \frac{2007}{2008} \right|$ 。求  $w$  的最小可能值。

Let  $x$  be a rational number and  $w = \left| x + \frac{2007}{2008} \right| + \left| x - \frac{2007}{2008} \right|$ . Find the smallest possible value of  $w$ .

9. 设  $m$  和  $n$  为正整数。已知表达式  $\left( \left( \left( (2)^2 \right)^{\cdot^{\cdot^{\cdot}}} \right)^2 \right) = \left( \left( \left( (4)^4 \right)^{\cdot^{\cdot^{\cdot}}} \right)^4 \right)$  含有  $m$  个 2 及  $n$  个 4。若  $k = \frac{m}{n}$ , 求  $k$  的值。

Let  $m$  and  $n$  be positive integers. Given that the number 2 appears  $m$  times and the number 4 appears

$n$  times in the expression  $\left( \left( \left( (2)^2 \right)^{\cdot^{\cdot^{\cdot}}} \right)^2 \right) = \left( \left( \left( (4)^4 \right)^{\cdot^{\cdot^{\cdot}}} \right)^4 \right)$ . If  $k = \frac{m}{n}$ , find the value of  $k$ .

10. 求  $\log_2(\sin^2 45^\circ) + \log_2(\cos^2 60^\circ) + \log_2(\tan^2 45^\circ)$  的值。

Find the value of  $\log_2(\sin^2 45^\circ) + \log_2(\cos^2 60^\circ) + \log_2(\tan^2 45^\circ)$ .